An Efficient Virtual Teeth modeling for dental training system

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Abstract

This paper describes an implementation of virtual teeth modeling for a haptic dental simulation. The system allows dental students to practice dental procedures with realistic tactual feelings. The system requires fast and stable haptic rendering and volume modeling techniques working on the virtual tooth. In our implementation, a volumetric implicit surface is used for intuitive shape modification without topological constraints and haptic rendering. The volumetric implicit surface is generated from input geometric model by using a closest point transformation algorithm. And for visual rendering, we apply an adaptive polygonization method to convert volumetric teeth model to geometric model. We improve our previous system using new octree design to save memory requirement while increase the performance and visual quality.

Key Words: Dental simulation, Haptic interaction, Volumetric implicit surface, BONO Octree

1. Introduction

For training, dental students usually use plastic teeth called the typodont model with real dental instruments to do some operations such as cavity preparation and other dental procedures. However, they can not feel the realistic tactile sensations due to the lack of detail needed to simulate accurately real teeth and procedures. Recently, virtual reality (VR) dental training systems have been introduced as alternative method to traditional training procedures. The systems allow dental students to experience virtual dental procedures with realistic tactile sensations in the same way they do in the real world.

Our dental training system requires two surface representations: volumetric and geometric surface representation. Geometric representation is effective for visual display while the volume-based approach is suitable for dental operations such as drilling and carving since it provides direct and intuitive modeling without topological constraints.

We use octree-based data structure to save geometric surface representation. However, our previous implementation [1] only works with the volume space whose resolutions are precisely $2^d \times 2^d \times 2^d$. Because of this limitation, it is difficult to increase visual quality of a generated object since it depends on the resolution of volume space. We implement a new octree design which can work with arbitrary

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resolution and also increase the performance of the system.

In this paper, we detail how to:

- generate volumetric implicit surface from input geometric model by using Mauch's fast closest point transform (CPT) algorithm [3].
- generate multi-resolution mesh from the volume model being modified using an adaptive polygonization method [4].
- save the volumetric implicit surface and generated mesh data into octree based data structure by using Branch-on-need Octree strategy [5] in order to reduce required memory.

Our dental training system is shown in Fig. 1.

![Image of dental training system](image)

**Fig. 1.** Dental training system we have developed. A user is performing dental operations.

In the section 2, we describe how to make the virtual teeth for dental training system including CPT implementation and multi-mesh generation. Branch-on-need octree strategy (BONO) implementation is discussed in the section 3. We discuss conclusions and future work in section 4.

## 2. Virtual Teeth Modeling

Our system is based on a hybrid surface representation [2] (a combination of geometric and implicit surface representation) for a given 3D teeth object. This approach takes advantages of both surface representations.

The system consists of three main steps as follows. Fig. 2 shows our system process.
1. Read the 3D geometric information from an input Virtual Reality Modeling Language (VRML) file and generate the volumetric implicit surface. In our system, a volume implicit surface is used for intuitive shape modification and haptic rendering.

2. For visual display, the volume implicit surface from previous step is converted back to a new 3D geometric model that represents geometry more efficiently compared with volumetric representation. And this geometric model data is stored on an octree based data structure.

3. In the last step, the system traverse the whole octree to get the geometric model information (multi-resolution mesh data) in every graphical frame for the visual rendering.

![Fig. 2. Virtual teeth modeling based on hybrid surface representation](image)

### 2.1 Volumetric implicit surface generation

Our haptic rendering algorithm uses the implicit representation of the external surface $S$ of an object defined by the following implicit equation:

$$ s = (x, y, z) \in \mathbb{R}^3 \mid f(x, y, z) = 0 $$  

where $f$ is the implicit function (also called potential) and $(x, y, z)$ is the coordinate of a point in 3D space.

If the potential value is 0, then the point $(x, y, z)$ is on the surface. The set of all point for which the potential value is 0 defines the implicit surface. If $(x, y, z) > 0$, the point $(x, y, z)$ is outside the object. Otherwise if the potential is negative, the point $(x, y, z)$ is inside the surface.

We apply Mauch's fast closest point transform algorithm [3] to create a volumetric implicit surface representation from a geometric model. Firstly, the algorithm subdivides the 3D space containing the geometric model into a regular 3D grid. After that, it computes the closest point to the surface and its distance at each grid point by efficiently solving the Eikonal equation using the method of characteristics.

In our system, only potential values close to the implicit surface are involved in the computation. So only potential values inside the close neighborhood of the surface ranging from -1 to 1
according to the proximity to the closest point on the surface are our concern. The potential values out of this range are not used for surface modeling and haptic rendering. Fig. 3 shows the volumetric representation generated from a geometric teeth model.

![Image of a volumetric teeth model generated from a geometric model by CPT algorithm. The potential values are sampled at each grid point (red points).](image)

**Fig. 3.** A volumetric teeth model is generated from a geometric model by CPT algorithm. The potential values are sampled at each grid point (red points)

### 2.2 Multi-resolution mesh generation

After computing the closest point to a surface and its distance at each grid point, we remove the upper part and lower part of the 3D regular grid in which the potential values are greater than 1 to save the memory. And then we apply an adaptive polygonization method suggested by Velho [4] to generate the mesh corresponding to each grid points. This algorithm consists of two main steps as follows:

1. **INITIALIZATION:** uses a uniform space decomposition to create the initial mesh
   - A uniform space is constructed from 8 adjoining grid points. If there are one point with negative potential (inside the surface) value and one point with positive potential value (outside the surface), then the implicit surface pass through this uniform space.
   - Subdivide the uniform space which contains a part of the implicit surface into six tetrahedrons and then generate a initial mesh for each tetrahedron that serves as the basis for adaptive refinement (Fig. 4).

2. **REFINEMENT:**
   - For each initial mesh, test the corresponding surface patch for flatness. To test the flatness of an edge, we calculate the dot product of two normal vectors at two vertices. Edges are FLAT if dot product greater than a predefined value.
   - If edges are not FLAT, then recursively subdivide the mesh at points which is projected onto the implicit surface from mid points of non-flat edges according to the surface curvature until the desired accuracy is achieved since the mid point may not be on the
Fig. 4. Multi-resolution mesh generation: Initial step. (a) Uniform decomposition (b) Generate triangles based on tetrahedron.

Fig. 5. Multi-resolution mesh generation: Refinement step. (a) Move the point after projection onto the implicit surface (b) Generation of multi-resolution mesh

The generated mesh effectively represents sharp edges with a smaller number of triangles and does not introduce cracks (Fig. 6)

3 Branch-on-need Octree Implementation

In the previous implementation, we use even-subdivision strategy (each dimension is subdivided exactly by two at every level) to build the octree and assume that the resolutions of the volume are precisely $2^d \times 2^d \times 2^d$. Therefore the previous implementation does not work with the volume which has arbitrary resolution in 3-dimensional space. And when you increase each dimension two times to increase the visual quality of the output model, the number of regular grid points will be eight times bigger.
In addition, the teeth model is not fit in the 3D regular grid because the height of the teeth model is much smaller than the width and the depth of the teeth. To solve this issue, we implement another octree design suggested by Jane Wilhems and Allen Van Gelder [5]. This octree design is called branch-on-need octree (BONO).

![Fig. 6. A virtual tooth model with multi-resolution mesh](image)

The main idea of the BONO strategy is that the "lower" subdivision in each branching direction always covers the largest possible exact power of two yielding a range of the form $2^k-1$. The BONO strategy generates smaller number of nodes compare to the number of nodes created by even-subdivision strategy. Comparison of numbers of generated octree nodes are showed in table 1. In addition, during the polygonization process and saving the mesh data on the octree, we save the pointer of the last added node temporarily so we can add a new node directly without traversing from the root. These techniques improve the performance of octree operations such as inserting, deleting, updating and searching.

<table>
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<tr>
<th>Even-subdivision</th>
<th>BONO</th>
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<tr>
<td>Volume</td>
<td>Nodes</td>
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<tr>
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<tr>
<td>256x256x256</td>
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Table 1. Comparison between previous approach and BONO algorithm in term of the number of generate nodes
4 Conclusions

Our new octree implementation based on BONO algorithm showed that it can yield improvement in performance and reduce memory requirement. The dental simulation system now can work with any practical volume space, as the system selects the optimal resolution for a 3D model instead of a 3D regular grid. And the visual quality of generated teeth model is also improved due to higher resolution of volume. We plan to enhance the visual quality of the teeth by using GPU programming to implement some effects on the surface of the teeth model.

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References